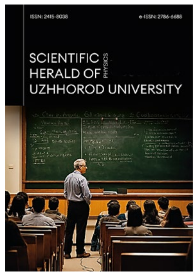
Scientific Herald of Uzhhorod University
Series "Physics"
Journal homepage: https://physics.uz.ua/en
Issue 55, 22–31
Received: 11.11.2023. Revised: 04.02.2024. Accepted: 28.02.2024



DOI: 10.54919/physics/55.2024.2psr2

The task of plotting flat cross-sections of polyhedrons as a means of developing divergent and algorithmic thinking and spatial representation ability

Duisebek N. Nurgabyl
Doctor of Physical and Mathematical Sciences, Professor
Zhetysu University named after I. Zhansugurov
040009, 187-a Zhansugurov Str., Taldykorgan, Republic of Kazakhstan
https://orcid.org/0000-0002-8111-862X

Kuanysh S. Nurpeissov
Lecturer
Zhetysu University named after I. Zhansugurov
040009, 187-a Zhansugurov Str., Taldykorgan, Republic of Kazakhstan
https://orcid.org/0000-0002-2786-7072

Abstract

Relevance. In this work, we developed problems on plotting the flat sections of polyhedrons in the case when the prism section is defined by the trace which is located on the plane of the prism base and does not have common points with the base of the prism, and by point, which belongs to the side rib of the prism.

Purpose. The purpose of the article is to establish that when solving problems on the construction of plane sections of polyhedra, students complete tasks; apply axioms and geometric properties; form and develop spatial representations; develop divergent and algorithmic thinking, the ability to reason logically, the ability to make correct arguments and conclusions.

Methodology. Problems on plotting the flat sections of polyhedrons using the internal design method are developed. To solve these problems, two types of projections are used: parallel and central. A flat section of a pentagonal prism is constructed using a parallel projection method. The central projection is used to construct the flat section of the pyramid.

Results. The designed tasks allow us to form and develop divergent and algorithmic thinking and form spatial representations.

Conclusions. After researching this topic, we came to the conclusion that the ability to solve problems of this type contributes to the formation and development of divergent and algorithmic thinking, the ability of spatial representation in future mathematics teachers.

Keywords: secant plane; flat section of a polyhedron; method of traces; parallel projection; central projection; divergent thinking.

Introduction

In the era of the development of scientific and technological progress in the everyday lives of people, various problems appear that require immediate solutions. To effectively solve these problems, it is necessary to develop mental abilities, which are essential factors of human thinking. While exploring the issues of thinking in the second half of the 20th century, the American

Suggested Citation:
Nurgabyl D, Nurpeissov K. The task of plotting flat cross-sections of polyhedrons as a means of developing divergent and algorithmic thinking and spatial representation ability. *Sci Herald Uzhhorod Univ Ser Phys.* 2024;(55):22–31. DOI: 10.54919/physics/55.2024.2psr2

*Corresponding author

psychologist J. P. Guilford [1] substantiated the theory of creativity. According to this theory, human thinking can be aimed at developing a unique solution to a problem using a strictly logically constructed algorithm. This process of thinking is called convergent thinking. On the other hand, according to this same theory, human thinking can be directed at considering a multitude of (multivariate) solutions to a problem. This approach to solving a problem is called divergent thinking.

Based on an analysis of the results of the international study of functional literacy of 15-year-old students of PISA-2018, the results of the international assessment of adult competencies of PIAAC and the national assessment of the quality of secondary and higher education in the Republic of Kazakhstan, the State Program for the Development of Education and Science of the Republic of Kazakhstan for 2020-2025 was adopted. The program is focused on solving key identified problems and developing a system of education and science to increase the country's competitiveness and provide approaches for the best practices of OECD countries [2]. Within the framework of this program, a number of priority tasks have been identified, of which we single out the following:

- developing the professional quality of teachers;
- providing training for competitive personnel;
- updating the content of secondary education.

The fulfilment of these tasks involves the active implementation of activity-based teaching methods, which contribute to the effective formation and development of the mental abilities of schoolchildren. The need for the formation and development of students' mental abilities, namely, divergent thinking, algorithmic thinking, and spatial representation ability, follows from the need to solve practical real-world problems. The importance of these abilities in connection with the development of modern management, logistics, information technology, and machine learning are constantly growing.

Consequently, teaching mathematics at a university should be oriented towards making future mathematics teachers ready to organize active activities to develop the divergent, algorithmic, and spatial thinking of schoolchildren [3; 4]. In addition, in the process of studying mathematics and information technology, students often have to face the fact that there they have poorly developed algorithmic thinking and an insufficient ability to find hidden spatial representations, and in the vast majority of cases, students lack the ability to construct simple algorithms and build spatial figures. Practice has shown that these problems, first of all, are the consequences of training being aimed only at the development of convergent thinking and the inefficient use of potential educational materials and educational tasks that form divergent and algorithmic thinking and spatial representation ability. This attitude of future mathematics teachers will ultimately lead to their mental abilities and readiness to effectively organize the thinking activities of schoolchildren remaining at poor levels and at the level of their learners. In this regard, the formation and development of divergent thinking, spatial representation and algorithmic thinking skills among future mathematics teachers is one of the main requirements for the successful preparation of future mathematics teachers [5-7].

Thought and spatial representation processes have been studied by many authors. In particular, a number of studies indicate the importance of divergent thinking in education, science, economics, and sociology. For example, W. I. O'Byrne, N. Radakovic and T. Hunter-Doniger [8] note that divergent thinking and creativity can play key roles in the formation of adaptive thinking and mathematical achievements and the creation of educational models in mathematical education. In their studies, V. K. Chumarina, N. V. Ilina and A. A. Prishchepa [9] use methods for developing divergent thinking as learning methods to increase the levels of self-identification and intercultural interaction among modern students. To create the concept of a multiple state, D. Cooper [10] uses divergent (multiple state) thinking as a tool for managing this state. N. Berlin, L. Tavani and M. Beasancon [11] investigated the relationship between students' performance and creativity based on assessments of creativity indicators and proved that verbal divergent thinking negatively predicts most of the relationships, while integrative thinking is positively correlated with scientific assessments. The issues of the formation and development of algorithmic and spatial thinking are separate self-dependent problems for research. For example, an article by R. M. Horbatiuk and V. V. Kabak [12] identifies the features of the formation of algorithmic thinking among future teachers engineers in the field of computer technology, which is an important intellectual component of their professional activity. In the article by T. Tikhonova [13], a method for using the agent created by Zvenigorodski as a tool for the formation of algorithmic thinking was developed. J. J. Foster, E. M. Bsales and E. Awh [14] investigated the effects of a latent spatial representation on the speed of visual information processing.

L. S. Bellmund, P. Gardenfors and E. I. Moser [15] state that the hippocampus is formed on the basis of a spatial representative sample that provides an oriented understanding of terrain in multiple cognitive spaces. The issues of organizing educational processes for the formation and development of thinking and spatial representation ability are closely related to the content of the educational materials of the disciplines. In this direction, studies have been carried out to develop the basic requirements for the content of academic disciplines in higher mathematics [16-18], which are offered to future mathematics teachers. The influence of the elementary beliefs, views and emotions of teachers on the formation of the content of university mathematics courses [19] is studied. A number of works [20-25] indicate that one of the methods for the formation and development of the research abilities of schoolchildren is to solve algebra and geometry problems in school. Undoubtedly, the most important aspect of such training is the scientific presentation of the educational material and the development of logical, algorithmic thinking and spatial representation abilities [26; 27].

However, these and other studies do not distinguish among the classes of problems that ensure the simultaneous formation and development of divergent and algorithmic thinking and spatial representation abilities in future mathematics teachers. The study showed that, in the sense of developing divergent thinking, spatial representation ability and algorithmic thinking, the search for multiple solutions and the construction of an algorithm to solve specific geometric construction problems are significant. This significance is clearly manifested in the construction of cross sections of polyhedra. Solving these construction tasks increases motivation and develops spatial representation ability, ingenuity, divergent and algorithmic thinking, graphical culture, and polytechnic skills. As is known, solving such problems includes the following stages: analysis, plotting, proof and research. Analysis and study of the formulation of a construction task contributes to the development of divergent and spatial thinking, and the construction and proof of the solution to the problem favour the development of algorithmic thinking [28-30]. Consequently, construction tasks play a key role in the formation of mental abilities and in the development of spatial representation ability in both teachers and pupils. V. A. Dalinger [31], G. D. Glazer [32], A. D. Semushin [33], N. F. Chetverukhin [34], A. R. Chernyaev [35], V. I. Butyrina [36], T. P. Pushkaryeva [37] and others have made significant contributions to teaching schoolchildren to solve geometric tasks on the construction of flat sections of polyhedra.

Materials and Methods

Let us present the problem proposed to future mathematics teachers who are 4th year students in the higher education institutions of the Republic of Kazakhstan in 2018: "On the side ribs of the cube ABCDA_1 B_1 C_1 D_1 are the points P,Q,R such that:

BP=1/3 BB_1, (1)

C_1 Q=2/3 CC_1, (2)

DR=1/3 AD. (3)

It A section of the cube with the ROM plane must be constructed". No student was able to correctly construct the desired cross section. In the same year, this task was proposed to young mathematics teachers at secondary schools and schools for gifted children. The result was the same. Analysis of the educational literature and the methods for teaching geometry at universities and schools showed that this result is a consequence of the fact that the training for solving problems on plotting the cross-sections of polyhedrons at the university and school levels was sporadic. In the future, although questions of constructing cross sections of polyhedrons will appear in some problems of geometry courses, university teachers and school teachers will mainly consider these tasks superficially. Practice shows that one of the main reasons for this negative attitude of mathematics teachers toward solving these types of problems is the lack of sufficient knowledge on this issue. Consequently, the formative and development capabilities needed to solve these tasks are practically not used by either university instructors or school teachers.

Thus, there are questions when searching for new approaches to teach various methods to solve problems on constructing flat sections of polyhedrons related to the formation and development of spatial representation ability and divergent and algorithmic thinking. One of the main approaches of this type of training is the design and solution of specially selected systems of exercises and tasks on constructing flat sections of polyhedrons. To solve this problem, we selected the following research methods:

theoretical: analysis of the psychological and pedagogical literature, laws, and regulatory documents related to the research problem and the study of the teaching methods of university teachers on the formation of professional qualities;

educational: methods for constructing flat sections of polyhedrons;

empirical: a survey of mathematics teachers at pedagogical universities and secondary schools in the Almaty region, written work to determine the knowledge and skills of future mathematics teachers on the construction of flat sections of polyhedral, and analysis of the results;

comparative analysis and mathematical processing of experimental data.

The first stage of the experimental work was carried out on the centre of excellence of the Almaty region to determine the level of the spatial representation ability and attitude to the study of the educational material "Construction of cross-sections of polyhedrons" among mathematics teachers (54 people) through questionnaires. An analysis of the survey results revealed that only less than 10% of the respondents were able to positively answer 50% or more of the posed questions. The results of the survey indicated a negative attitude of mathematics teachers to the lessons in the educational material "Construction of sections of polyhedrons" and a low level of knowledge and skills on the methods for constructing flat sections of polyhedrons. To determine the level of divergent and algorithmic thinking in future mathematics teachers, written works were developed. The written works were performed by students of the "mathematics" specialty (31 students). In control paper No. 1, five tasks were proposed to determine the level of students' divergent thinking. In written work 2, students were presented with five tasks to determine the level of their algorithmic thinking. In control work No. 3, students were presented a simple task on plotting a flat cross-section of a cube. Analysis of the data obtained from the results of control work No. 1 revealed that only 17% of the respondents were showed an average level of divergent thinking, and the rest of the students showed a low level of divergent thinking. Analysis of the results of control work No. 2 revealed that only 25% of students demonstrated an average level of algorithmic thinking, and the remaining students showed low levels of algorithmic thinking. The results of control work No. 3 showed that all students, without exception, failed to correctly plot the flat cross-section of a cube.

In the next stage, the directions for designing problems to construct a flat section of a polyhedron and developing methods for solving these problems as well as approaches to the formation of spatial representation ability and divergent and convergent thinking in students were identified The aim of this work is to develop a methodology for teaching future mathematics teachers how to build flat sections of polyhedra using the simplest property of stereometry, the traces method, and the internal design method and determining how to help them develop their spatial representation ability and divergent and algorithmic thinking in the context of the updated educational content in the Republic of Kazakhstan.

Results and Discussion

Development tasks on plotting flat sections of polyhedrons that contribute to the formation and development of divergent thinking and spatial representation ability. Here, we construct problems and consider questions that will develop the spatial representation ability and divergent thinking of students using the example of solving problems devoted to the construction of a flat section of a polyhedron. Problem 1. On the edges of the cube ABCDA_1 B_1 C_1 D_1, different points M,N,K are given. A flat section of the cube secant by the plane MNK must be constructed. After a careful analysis of the conditions of problem 1, students will be convinced that the points M,N,K can be placed on the ribs of the cube ABCDA_1 B_1 C_1 D_1 in multiple ways. For example, the following solutions exist:

points M,N,K can be located on adjacent parallel ribs, such as on ribs M∈AA_1,N∈BB_1,K∈CC_1;

two points, such as M and N, can be located on adjacent parallel ribs, such as M∈AA_1,N∈BB_1, and the third point can be located non-contiguous with these ribs, such as on K∈DC;

points M,N,Kcan be located on adjacent non-parallel edges, such as on edges M∈AA_1,N∈A_1 B_1,K∈AD;

points M,N,K can be located on the same rib.

Problem 2. Two points M and N are given on the ribs of the cube ABCDA_1 B_1 C_1 D_1 and the third point K is located inside some face of the given pyramid. A flat section of the cube formed by the planeMNK must be constructed. Analysing the conditions of this problem, the students should be sure that the points M,N,Kcan be arranged in multiple ways for the given cube ABCDA_1 B_1 C_1 D_1. Thus, analysis of the conditions and development of a plan to solves these or similar problems make it possible to formulate and develop divergent thinking and a spatial representation ability in both students and schoolchildren. Development and solution of tasks on plotting a flat cross-section of a polyhedron using the simplest properties of straight lines and planes. In this subsection of the article, when constructing flat cross-sections of polyhedrons, we only use the simplest properties of straight lines and planes. Namely, one and only one straight line can be drawn through two given points, or through a point located outside of a given straight line, one and only one straight line can be drawn parallel to this straight line. In addition, we assume that the plane is constructed if three points are given that do not lie on the same straight line, a straight line and a point outside this straight line are given, or parallel straight lines are given.

It is known that the plane that intersects a given polyhedron is called the secant plane [38; 39]. The secant plane intersects the faces of the polyhedron along the segments [40; 41]. Therefore, a polygon is obtained, which is called the flat section of the polyhedron. Here, we consider only some on the construction tasks or rather tasks on constructing flat sections for some polyhedrons.

Problem 3. Construct a plane section of the parallelepiped ABCDA_1 B_1 C_1 D_1 at a secant plane π passing through the given points M,N,K (Fig. 1).

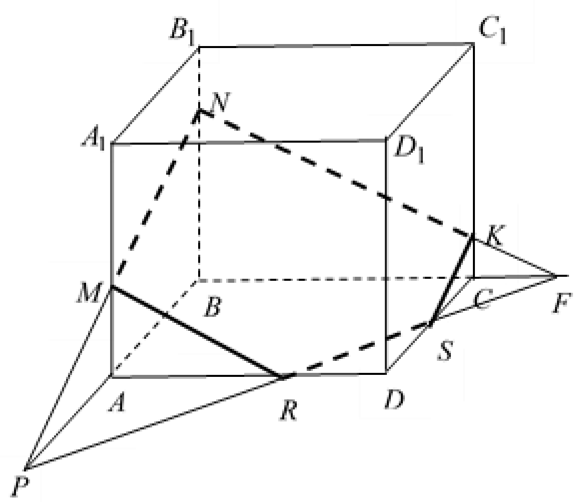


Figure 1. The parallelepiped ABCDA_1 B_1 C_1 D_1

Decision. We begin to search for a solution to the task by performing an analysis. We conditionally assume that the problem is solved. The points M,N,K are located on different ribs. Let us depict an imaginary figure defined by these elements of the desired section. We reveal the features of these elements. Then, together with students, we answer the following questions:

where can the vertices of the desired figure be located?

what geometric shape can the drawn section determine?

how can the geometric shape be used to find other peaks of the desired figure?

how can the geometric shape build the desired cross-section?

Then, we determine the construction sequence. In other words, to construct the desired section, we use the following algorithm:

1) consider face AA_1 B_1 B of this prism. On this face lie the points M and N of the desired section. We plot a straight passing through points M and N. We obtain an image MN secant plane π on face AA_1 B_1 B;

2) we plot the straight line AB to intersect with the straight line MN. The intersection point is denoted as P;

3) consider the B_1 BCC_1 face of the prism under consideration. On this face lie the points K and N of the desired section. We plot a straight line NK. We obtain an image NK of the secant plane π on face CC_1 B_1 B;

4) we plot the straight line BC to intersect with the straight line NK. The intersection point is denoted as F;

5) points F and P belong to the plane ABCD. We plot the straight line FP. The straight line FP intersects rib AD at point R and rib DC at point S. We obtain an image of the secant plane π on face ABCD of the given prism ABCDA_1 B_1 C_1 D_1;

6) points M and R belong to face AA_1 D_1 D. By connecting points M and R, we obtain an image MR of the secant plane π on face AA_1 D_1 D;

7) points S and K belong are on face D_1 DCC_1. By connecting pointsS and K, we obtain an image SK of the secant plane π on face D_1 DCC_1.

Thus, MNKSR is the desired flat cross-section of the parallelepiped ABCDA_1 B_1 C_1 D_1 on the secant plane π passing through points M,N,K. Based on the meaning of the problem, points M, N, and K do not lie on one straight line. Therefore, the task has a unique solution. After constructing the desired cross-section, students need to ask the following questions:

under what conditions does this task have a solution?

under what conditions does this task have no solution?

for what data does the task in question have several solutions?

The answers to these questions provide an opportunity to research the task under consideration. Thus, we are convinced that the proposed algorithm for constructing a polyhedron section improves students' algorithmic thinking and provides an understanding of the method to solve problems on the construction of a cross-section of a polyhedron. Development and solution of tasks on plotting a flat cross-section of a polyhedron using the traces method. In this subsection, we construct a problem on constructing a section of a given polyhedron using a plane that is defined by a point located on the surface of the given polyhedron and a given trace that is located on the plane at the base of the polyhedron and does not have common points with the base of the polyhedron. Thus, we construct and solve the problem on constructing a flat section of a polyhedron using the trace method.

Problem 4. Construct a flat cross-section of the pentagonal straight prism ABCDEA_1 B_1 C_1 D_1 E_1 with the secant plane π. The plane π is given by point K, which belongs to side rib CC_1 and trace l, which is located on plane ABCDE and does not have common points with the base of the prism but has a common intersection point with the continuation of each rib of the base of the prism. Analysis. Suppose that polygon KMNPR is the desired section (Fig. 2). To construct the desired section KMNPR, it is sufficient to construct its vertices, M,N,P,R, which are the intersection points of the secant plane π with the corresponding edges DD_1,EE_1,AA_1,BB_1, of prism ABCDEA_1 B_1 C_1 D_1 E_1.

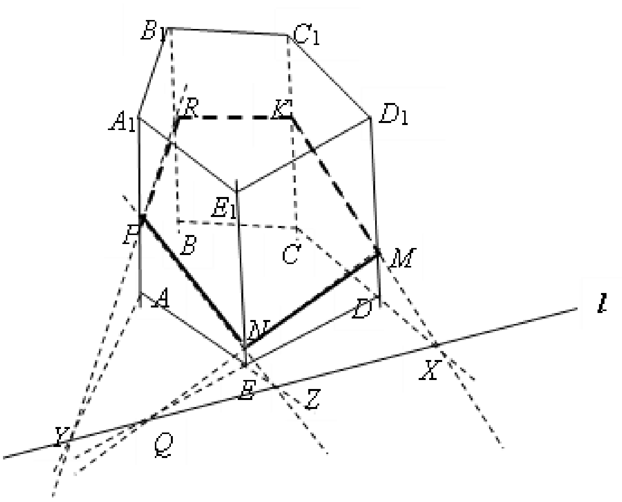


Figure 2. Construct a flat cross-section of the pentagonal straight prism ABCDEA_1 B_1 C_1 D_1 E_1

To construct point M=π∩DD_1, it is sufficient to construct a straight line that is determined by the intersection of the given secant plane π with face DCC_1 D_1. To construct this line, it is sufficient to build another point belonging to the secant plane π in the plane of this face. How can we find an image of such a point? Since the straight line l lies in the plane of the base of the prism, it can intersect the plane of face CDD_1 C_1 only at a point that belongs to the straight line CD, i.e., at point X=CD∩l. Points K and X belong to the same plane. From here, we determine point M=KX∩DD_1. To construct other vertices, it is sufficient to construct points Y,Q,Z, which belong to straight line l and the corresponding faces ABB_1 A_1, DEE_1 D_1, andAEE_1 A_1. Thus, the problem of constructing a section of a given polyhedron is solvable. Plotting:

1. As already noted, this trace l and straight line CD lie on the plane of the base of the prism. Then, we can determine the intersection point of trace l and straight line CD. We denote this point as X=l∩CD, where X∈π.

2. Since points X and K belong on the same plane face DCC_1 D_1, we can draw a straight line XK belonging to the secant plane π. The straight line XK intersects rib DD_1 at some point M=π∩DD_1.

3. The trace l and straight line ED lie on the plane of the base of the prism. Then, we can determine the intersection point of trace l and straight line ED. This point is denoted as Q=l∩ED, which belongs to the secant plane π.

4. Since points Q and M belong on the same plane of face DEE_1 D_1, we can draw a line QM belonging to the secant plane π. The straight line QM intersects edge EE_1 at some point N=π∩EE_1, where N∈π.

5. Given that trace l and line AE lie on the plane of the base of the prism, we can then determine the intersection point of trace l and straight lineAE. This point is denoted as Z=l∩AE, which belongs to the secant plane π.

6. Points Z and N are located on the same plane of faceAEE_1 A_1 and belong to the secant plane π. Then, we can draw a straight line ZN belonging to the secant plane π. Line ZN intersects edge AA_1 at the point P=π∩AA_1.

7. Trace l and straight line AB lie on the plane of the base of the prism. Then, we can determine the intersection point of trace l and straight line AB. This point is denoted as Y=l∩AB, which belongs to the secant plane π.

8. Points P and Y belong on the same plane of face ABB_1 A_1, and they belong to the secant plane π. Then, we can draw straight line PY belonging to the secant plane π. Straight line PY intersects edge BB_1 at point R=π∩BB_1, which belongs to the secant plane π.

9. By connecting points K and M, M; N, N and P; P and R; and R and K, we obtain the geometric figure KMNPR.

Proof. Because trace l and straight lines BA,CD,DE,AE belong to the plane of the base of the prism, we can uniquely determine the points Y=l∩BA,X=l∩CD,Q=l∩DE,Z=l∩AE. Then, successively we obtain the following.

1. K∈π, and X∈π ⇒KX∈π. Therefore, KX∩DD_1=M∈π. From here, KM∈π.

2. M∈π, and Q∈π ⇒QM∈π. Hence, we find thatMQ∩EE_1=N∈π. Then, MN∈π.

3.N∈π, Z∈π ⇒NZ∈π, and NZ belongs to the plane of face AEE_1 A_1. Therefore, NZ∩AA_1=P∈π. From here, PN∈π.

4. $P \in \pi$, and $Y \in \pi \Rightarrow PY \in \pi$. In addition, the straight line PY also belongs to the plane of face ABB_1A_1. Hence, we find that $PY \cap BB_1 = R \in \pi$. Then, $RP \in \pi$.

5. Therefore, KMNPR is the desired cross section.

Research. Trace l of the desired secant plane π is located on the plane of the base of the given prism; however, it does not have common points with the base of the prism. In addition, point K belongs to the secant plane and side rib CC_1 of the prism. Therefore, the secant plane π defined by point K and trace l intersects the side ribs (or their extensions) at points K,M,N,P,R of the prism under consideration. Thus, the intersection points of the secant plane with a given prism exist. On the other hand, $K \notin l$. Then, point K and trace l uniquely determine the secant plane π. Therefore, the problem under consideration has a unique solution. The creation and solution of problems on the construction of sections of polyhedrons using the internal design method. The internal design method is conditionally divided into two types of designs: parallel design and central design methods. 1. Consider a parallel design method for constructing the flat section of a prism.

Problem 5. Construct a flat cross-section of prism $ABCDA_1B_1C_1D_1$ with plane π defined by the interior points $M \in AA_1, N \in B_1C_1$ and by point K located on the inside section of face CDD_1C_1.

Plotting and proof:

1) we define the projection of point K in the prism $ABCDA_1B_1C_1D_1$ (Fig. 3);

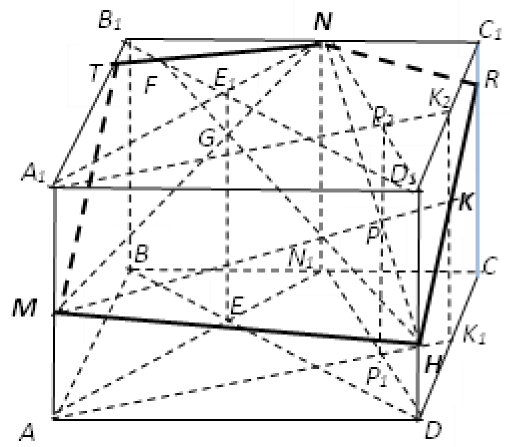


Figure 3. The projection of point K in the prism $ABCDA_1B_1C_1D_1$

To do this, draw a segment K_1K_2 so that $K_1K_2 \parallel CC_1$ ┤ and $K \in K_1K_2$;

2) we draw segment MK, where $MK \in \pi$, since $MK \in \pi$, and $K \in \pi$;

3) we find the projections A_1K_2 and AK_1 of segment MK on planes ABCD and $A_1B_1C_1D_1$, respectively;

4) we build segments A_1K_2 and AK_1;

5) points P_2 and P_1 are common for planes $A_1K_2K_1A$ and DD_1NN_1. Therefore, these planes intersect along the straight line P_1P_2. Then, there exists a point $P = MK \cap P_1P_2$ where $P \in \pi$;

6) by connecting points P and N, we find that point H is the intersection point of NP and DD_1. Since $P \in \pi$ and $N \in \pi$, we obtain $H \in \pi$;

7) we draw a segment passing through points $H \in \pi$ and $K \in \pi$ to intersect with rib C_1C. The intersection point is denoted as R. Obviously, $HR \in \pi$;

8) connect points $R \in \pi$ and $N \in \pi$. We obtain $RN \in \pi$;

9) we draw planes A_1NN_1A and B_1BDD_1. These planes intersect along the straight line E_1E;

10) we find point G at which the straight lines $MN \in \pi$ and E_1E intersect, where $G \in \pi$;

11) we build a straight line $HG \in \pi$;

12) we find point $F \in \pi$ at which the straight line $HG \in \pi$ intersects the straight line B_1D_1;

13) we find point T at which the straight lines $NF \in \pi$ and A_1B_1 intersect, where $T \in \pi$;

14) we connect points $T \in \pi$ and $M \in \pi$, $M \in \pi$ and $H \in \pi$. We obtain segments $TM \in \pi$ and $MH \in \pi$;

15) the resulting polygon MHRNT is the desired flat cross-section.

Proof. Since the points do not lie on one straight line, the problem has a unique solution. The process of constructing a flat section of a polyhedron using the parallel design method proves that the proposed methodology for constructing flat sections of a polyhedron allows one to efficiently form and develop the spatial representation ability and convergent and algorithmic thinking in future mathematics teachers. 2. Now consider the central design method for the construction of flat cross-sections of a pyramid.

Problem 6. Construct a section of quadrangular pyramid ABCDS with plane π defined by the internal points $M \in SC, N \in SB$ and point K located on the inside of the face ASD. Solution:

1. We define the central projection of the straight line $KN \in \pi$ on the plane of base ABCD of the pyramid ABCDS (Fig. 4).

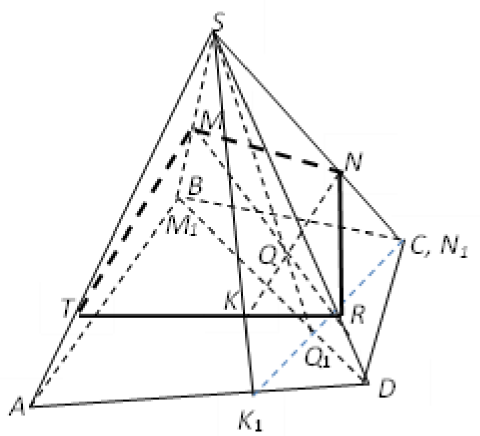


Figure 4. The central projection of the straight line $KN \in \pi$ on the plane of base ABCD

To this end, we draw a straight SK on the plane of face ADS. Then, we obtain the projection $K_1 = SK \cap AD$ of point K on rib AD. The central projection of point N is point $N_1 = C$. By connecting the points K_1 and N_1, we obtain the projection K_1N_1 of the straight line $KN \in \pi$ on the base plane ABCD.

2. The central projection of point M is point $M_1 = B$. The central projection of any point of rib SD is point D. By connecting points B and D, we obtain the plane BDS.

3. The planes BDS and K_1 N_1 S intersect along the straight line SQ_1. Since SQ_1 and KN∈π belong to the same plane K_1 N_1 S, the line SQ_1 intersects the straight line KN∈π at some point Q∈π.

4. Points M∈π and Q∈π belong to the same plane BDS, and therefore, the line MQ∈π intersects the rib SD at some point R∈π.

5. By drawing a straight line through points R∈π andK∈π, we obtain point T∈π.

6. By connecting points R∈π and T∈π, T∈π and M∈π, M∈π and N∈π, and N∈π and R∈π, we obtain RTMN, which is the desired flat cross-section. According to the meaning of the problem, points M,N,K do not lie on one straight line. Therefore, problem 6 has a unique solution.

Thus, we proposed methods for constructing sections of polyhedrons. This approach for teaching students greatly facilitates the process of students mastering solutions to problems on constructing flat cross-sections of polyhedrons. A phased review of the solutions to these types of problems and recognizing logical connections is accompanied by the construction of the elements of the desired section, which form the basis for compiling the sequence of the algorithm for solving problems. The proposed tasks ensure the formation and development of divergent and algorithmic thinking, spatial representation ability, professional knowledge, and understanding the meaning of construction tasks, allowing them to be easily understood and solved [42; 43]. For the created problems, construction of a flat section was performed by following the main stages of solving problems: analysis, construction, proof, and research. The obtained results predetermine the direction for further research in the theory of designing the content of academic disciplines, and they are of great applied importance in the process of forming the professional qualities of future mathematics teachers.

Note that the complexity of the widespread use of teaching methods for future mathematics teachers for constructing cross-sections of polyhedrons is the cumbersomeness of their construction and the lack of systems for solving these problems in academic geometry courses, which are necessary for the formation of mathematical thinking and developing the skills to solve such problems in students and schoolchildren.

Conclusions

The final work of this research was the implementation of the diagnostic measure, the aim of which was to prove the effectiveness of the developed methodology for constructing flat sections of polyhedrons. The ability to solve this type of problem contributes to the formation and development of divergent and algorithmic thinking and of spatial representation ability in future mathematics teachers. With the aim to identify the level of formed divergent and algorithmic thinking, the ability to solve problems on constructing a flat section of a polyhedron in future mathematics teachers was assessed using control work No. 4. Analysis of the results of control work No. 4 revealed that 85% of students could correctly and reasonably construct a flat cross-section of a polyhedron using simple geometric (stereometric) properties, 67% of the students were able to construct a flat cross-section using the traces method, and 53% were able to construct a flat cross-section using the internal design method. When solving the problems of this control work, 71% of respondents showed a high level of divergent thinking, and the remaining 29% showed a medium level of divergent thinking. Sixty seven percent of students showed a high level of algorithmic thinking, and 15% of respondents showed that they had a low level of algorithmic thinking. The obtained results prove the effectiveness of the proposed methodology for constructing flat sections of polyhedrons in the context of the formation and development of high-quality professional knowledge, spatial representation ability, and divergent and algorithmic thinking in future mathematics teachers.

Any opinions expressed and the conclusions and recommendations of the authors do not reflect the views of the Ministry of Education and Science of the Republic of Kazakhstan.

Acknowledgements

This research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19676696).

Conflict of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

1. Guilford JP. *Intelligence, creativity their educational implications*. San Diego: R. R. Knapp; 1968.
2. Hartz C, Mazurek A, Miki M. The Application of 2D and 3D Graphic Statics in Design. *J Int Associat Shell Spatial Struct.* 2018;59(4):235-242**.** DOI: 10.20898/ j.iass.2018.198.032
3. Kenzhebekova RI, Kozhadeldiyeva SS, Moldabek K, Rizaeva LA, Kazybayeva KU. Formation of Learning Research Skills through Solving Arithmetic Problems. *Syst Rev Pharm.* 2020;11(10):698-705. DOI: 10.31838/srp.2020.10.103.
4. Abylkassymova AE, Kalybekova ZA, Zhadrayeva LU, Tuyakov YA, Iliyassova GB. Theoretical foundations of the professional direction of teaching mathematics course in higher educational institutions. *Glob Stoch Anal.* 2021;8(2):311-322.
5. Marzhan D, Maxat D, Akbota A, Moldabek K, Rabiga K, Rysbayeva G. The development of computational skills of visually impaired children of primary classes. *Cypr J Educ Sci.* 2022;17(2):451-463. DOI: 10.18844/CJES.V17I2.6848.
6. Rehman HU, Darus M, Salah J. Generalizing Certain Analytic Functions Correlative to the n -th Coefficient of Certain Class of Bi-Univalent Functions. *J Math.* 2021;2021:6621315. DOI: 10.1155/2021/6621315.

7. Salah J, Ur Rehman H, Al-Buwaiqi I. The Non-Trivial Zeros of The Riemann Zeta Function through Taylor Series Expansion and Incomplete Gamma Function. *Math Statist.* 2022;10(2):410-418. DOI: 10.13189/ms.2022.100216.
8. O'Byrne WI, Radakovic N, Hunter-Doniger T. Designing Spaces for Creativity and Divergent Thinking: Pre-Service Teachers Creating Stop Motion Animation on Tablets. *Int J educ Math sci Tech.* 2018;6(2):182-199. DOI: 10.18404/ ijemst.408942
9. Chumarina VK, Ilina NV, Prishchepa AA. Designing of Holidays as an Effective method of ethnic cultural influencing on the mentality of modern students which learn fine arts in universities. *Turk Online J Design Art Commun*. 2017;7:1587-1596. DOI: 10.7456/1070DSE/135
10. Cooper D. Prefiguring the State. *Antipode.* 2017;49(2):335-356. DOI: 10.1111/anti.12277
11. Berlin N, Tavani J-L, Beasancon M. An exploratory study of creativity, personality and schooling achievement. *Educ Econ*. 2016;24(5):536-556. DOI: 10.1080/09645292.2015.1117580
12. Horbatiuk RM, Kabak VV. The use an Information System Algostudy for Algorithmic thinking formation of Future Engineer-Teachers in the Field of Computer technologies. *Info Tech Learn Tool.* 2019;69(1):124-138. DOI: 10.33407/itlt.v69i1.2385
13. Tikhonova T. *Agents as a Tool for the Formation of Algorithmic Thinking of Schoolchildren*; 2017. https://volconf.ru/files/archive/02_30.03.2016.pdf
14. Foster JJ, Bsales EM, Awh E. Covert Spatial Attention Speeds Target Individuation. *J Neurosci.* 2020;40(13):2717-2726. DOI: 10.1523/JNEUROSCI.2962-19.2020
15. Bellmund JLS, Gardenfors P, Moser EI. Navigating cognition: Spatial codes for human thinking. *Sci.* 2018;362(6415):eaat6766. DOI: 10.1126/science.aat6766.
16. Bekish U, Nurgabyl D, Abdoldinova G, Tulegulov D, Basheyeva Zh. Methodology of Designing the Pithy Components of Elective Courses on the Higher Mathematics of Pedagogical Profile. *Int J Engineer Res Tech.* 2020;13(3):407-4013.
17. Dzhomartova SA, Mazakov TZ, Karymsakova NT, Zhaydarova AM. Comparison of two interval arithmetic. *Appl Math Sci.* 2014;69-72:3593-3598. DOI: 10.12988/ams.2014.44301.
18. Mazakov T, Wójcik W, Jomartova S, Karymsakova N, Ziyatbekova G, Tursynbai A. The stability interval of the set of linear system. *Int J Electr Telecom.* 2021;67(2):55-161. DOI: 10.24425/ijet.2021.135958.
19. Hart LC, Auslander SS, Venuto N. A review of research on effect of elementary prospective teachers in university mathematics content courses 1990-2016. *Sch Sci Math.* 2019;119(1):3-13. DOI:10.1111/ssm.12310\
20. Kolyagin YuM. *Methodology of teaching mathematics in middle school. Private of methodology*. Moscow: Obrazovaniye; 1977.
21. Episheva OB. *General methods of teaching mathematics in high school: Lecture course*. Tobolsk: Publishing house of TSPI after D.I. Mendeleev; 1997.
22. Salah J, Al Hashmi M, Rehman HU, Al Mashrafi K. Modified Mathematical Models in Biology by the Means of Caputo Derivative of a Function with Respect to Another Exponential Function. *Math Statist.* 2022;10(6):1194-1205. DOI: 10.13189/ms.2022.100605.
23. Salah J, Rehman HU, Al Buwaiqi I, Al Azab A, Al Hashmi M. Subclasses of spiral-like functions associated with the modified Caputo's derivative operator. *AIMS Math.* 2023;8(8):18474-18490. DOI: 10.3934/math.2023939
24. Cherkasov PC. *Methodology of teaching mathematics in middle school. General of methodology*. Moscow: Obrazovaniye; 1985.
25. Mishin VI. *Methodology of teaching mathematics in middle school. Private of methodology*. Moscow: Obrazovaniye; 1987.
26. Chernozub A, Korobeynikov G, Mytskan B, Korobeinikova L, Cynarski WJ. Modelling mixed martial arts power training needs depending on the predominance of the strike or Wrestling fighting style. *Ido Movem Cult.* 2018;18(3):28-36. DOI: 10.14589/ido.18.3.5.
27. Tashimbetova A, Rysbaeva A, Suleimenov Z, Kalybekova Z, Sydykova D. Clusters in the gas dynamics and mathematical modeling in mathcade the results of the study. *Int J Engin Technol (UAE).* 2018;7(3.15 Special Issue 15):321-323. DOI: 10.14419/ijet.v7i2.29.13646.
28. Luiza R, Rabiga K, Amina A, Borashkyzy AU, Uaidullakyzy E, Bakhytgul S. Formation of research skills of students through solving problems in teaching mathematics in primary classes. *Cypr J Educ Sci.* 2022;17(8):2567-2579. DOI: 10.18844/cjes.v17i8.7824.
29. Kondratenko YP, Klymenko LP, Sidenko IV. Comparative analysis of evaluation algorithms for decision-making in transport logistics. *Stud Fuzz Soft Comp.* 2014;312:203-217. DOI: 10.1007/978-3-319-03674-8_20.
30. Kondratenko YP, Encheva SB, Sidenko EV. Synthesis of intelligent decision support systems for transport logistics. *Proceed 6th IEEE Int Conf Intell Data Acquis Adv Comp Syst: Technol Applic, IDAACS'2011.* 2011;2:642-646. DOI: 10.1109/IDAACS.2011.6072847.
31. Dalinger VA. *Methods of teaching students' stereometry through problem tasks*. Omsk: Publishing House of OmGPU; 2001.
32. Glazer GD. *The development of spatial representations of schoolboy when learning geometry*. Moscow: Pedagogika; 1978.
33. Semushin AD. *Methods of teaching geometric constructions in the course of stereometry.* Moscow: Publishing House of the APN of the RSFSR; 1952.
34. Chetverukhin NF. *Stereometric tasks in a projection drawing*. Moscow: Uchpedgiz; 1952.

35. Chernyaev AR. *Tasks for constructing cross sections of polyhedra as a means of developing spatial thinking in the course of geometry*. Omsk: Publishing House of OmGPU; 2003.
36. Butyrina VI. Training in the construction of sections as a means of developing spatial representation at the lessons of stereometry. *Sci Sch*. 2012;2012(3):86-89.
37. Pushkaryeva TP, Stepanova TA, Kalitina VV. Didactic tools for the Students' Algorithmic Thinking development. *Educ Sci.* 2017;19(9):126-143. DOI: 10.17853/1994-5639-2017-9-126-143.
38. Mikhailov P, Ualiyev Z, Kabdoldina A, Smailov N, Khikmetov A, Malikova F. Multifunctional Fiberoptic Sensors For Space Infrastructure. *East-Eur J Enterp Technol.* 2021;5(5-113):80-89. DOI: 10.15587/1729-4061.2021.242995.
39. Salah JY. Properties of the modified caputo's derivative operator for certain analytic functions. *Int J Pure Appl Math.* 2016;109(3):665-671. DOI: 10.12732/ijpam.v109i3.14.
40. Mustafin AT. Synchronous oscillations of two populations of different species linked via interspecific interference competition. *Izv Vyssh Uchebn Zaved. Prikl Nelin Dinam.* 2015;23(4):3-23. DOI: 10.18500/0869-6632-2015-23-4-3-23.
41. Bairbekova G, Mazakov T, Djomartova S, Nugmanova S. Interval arithmetic in calculations. *Open Engin.* 2016;6(1):259-263. DOI: 10.1515/eng-2016-0036.
42. Mazakov TZ, Jomartova SA, Shormanov TS, Ziyatbekova GZ, Amirkhanov BS, Kisala P. The image processing algorithms for biometric identification by fingerprints. *News Nat Acad Sci Rep Kazakhstan, Ser Geol Techn Sci.* 2020;1(439):14-22. DOI: 10.32014/2020.2518-170X.2.
43. Orazbayev B, Kozhakhmetova D, Orazbayeva K, Utenova B. Approach to modeling and control of operational modes for chemical and engineering system based on various information. *Appl Math Inform Sci.* 2020;14(4):547-556. DOI: 10.18576/AMIS/140403.

Задача на побудову плоских перерізів многогранників як засіб розвитку дивергентного та алгоритмічного мислення і здатності до просторового уявлення

Дуйсебек Н. Нургабил
Доктор фізико-математичних наук, професор
Жетисуський університет імені І. Жансугурова
040009, вул. Жансугурова, 187-а, м. Талдикорган, Республіка Казахстан
https://orcid.org/0000-0002-8111-862X

Куаниш С. Нурпеісов
Викладач
Жетисуський університет імені І. Жансугурова
040009, вул. Жансугурова, 187-а, м. Талдикорган, Республіка Казахстан
https://orcid.org/0000-0002-2786-7072

Анотація

Актуальність. У роботі розроблено задачі на побудову плоских перерізів многогранників у випадку, коли переріз призми задається слідом, який лежить у площині основи призми і не має спільних точок з основою призми, та точкою, яка належить бічному ребру призми.

Мета. Мета статті - встановити, що під час розв'язування задач на побудову плоских перерізів многогранників учні виконують завдання; застосовують аксіоми та геометричні властивості; формують і розвивають просторові уявлення; розвивають дивергентне та алгоритмічне мислення, вміння логічно міркувати, вміння робити правильні аргументи та висновки.

Методика. Розроблено задачі на побудову плоских перерізів многогранників методом внутрішнього конструювання. Для розв'язування цих задач використовуються два види проекцій: паралельна та центральна. За допомогою методу паралельного проектування побудовано плоске переріз п'ятикутної призми. Центральне проекціювання використано для побудови плоского перерізу піраміди.

Результати. Розроблені завдання дозволяють формувати та розвивати дивергентне та алгоритмічне мислення, формувати просторові уявлення.

Висновки. Дослідивши дану тему, ми дійшли висновку, що вміння розв'язувати задачі такого типу сприяє формуванню та розвитку дивергентного та алгоритмічного мислення, здатності просторового уявлення у майбутніх вчителів математики.

Ключові слова: Січна площина; плоский переріз многогранника; метод слідів; паралельне проектування; центральне проектування; дивергентне мислення.